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UDC 534

FORCED OSCILLATIONS WITH A SLIDING REGIME RANGE OF A TWO-MASS SYSTEM INTERACTING WITH A FIXED STOP

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We investigate, by the method developed in [1], the forced oscillations with a sliding regime range of a two-mass system with elastic connection between the elements, impacting a fixed stop. The system being considered is a dynamic model for a number of vibrational mechanisms. Forced oscillations with a sliding regime range of a system with shock interactions are periodic motions accompanied by a period of an infinite succession of instantaneous collisions of two fixed elements of the model [2]. Within the framework of conditions of roughness of the parameter space [3], in this paper we study by the method of [1] periodic motions with a sliding regime range of a two-mass system with a stop. This problem was posed because in real systems the velocity recovery factor R changes from shock to shock, mainly taking small values (0, 0.2). At the same time, the regions of realizability of one-impact oscillations, in practice the most essential ones among motions with a finite number of interactions over a period, narrow down sharply as R decreases and becomes very small even for $R < 0.6$ [4]. Thus, the stability of the given operation can be ensured by a law of motion which is independent or weakly dependent on R (*) (see footnote on the next page). By virtue of what has been said above,

finite-impact periodic modes are little suitable for this purpose. Regions, delineated in the parameter space of the model being considered, of existence of stable periodic motions with a sliding regime range have proved to be sufficiently broad. By virtue of the adopted approximation of the sliding regime, the dynamic characteristics of these motions do not depend upon R. The circumstances mentioned confirm the practical value of motions with a sliding regime range in dynamic systems with impact interactions.

1. Equations of motion and point mapping. The model we have chosen to investigate is a system of two masses m_1 and m_2 joined by a spring with a linear characteristic c. On each mass there acts a constant force P_1 and P_2, respectively. The motion of mass m_1 is restricted by a fixed barrier on contact with which a shock interaction takes place, characterized by the Newton hypothesis. Collisions between mass m_2, which is subject to the influence of an external harmonic force $F \sin \Omega t$, and mass m_1, as well as the fixed barrier, do not occur. The displacements η of mass m_1 and ζ of mass m_2 are measured from the surface of the stop in the one direction such that $\eta > 0$ in the time interval between collisions in the system. We introduce the dimensionless variables and parameters

$$y = m_2\Omega^2\eta \, / \, F, \quad x = m_2\Omega^2 \, (\zeta + P_2c^{-1}) \, / \, F, \quad \tau = \Omega t$$
$$\nu^2 = c \, / \, m_2\Omega^2, \qquad P = (P_1 + P_2) \, / \, F, \qquad \mu = m_1 \, / \, m_2$$

Then under assumptions usual to the given class of problems the behavior of the model being examined is described by the following equations:

$$\mu y^{\cdot\cdot} = \nu^2 (x - y) - P, \quad x^{\cdot\cdot} = \nu^2 (y - x) + \sin \tau, \quad y > 0 \tag{1.1}$$

$$y_+^{\cdot} = -Ry_-^{\cdot}, \quad y = 0 \tag{1.2}$$

$$y = y^{\cdot} = 0, \quad x^{\cdot\cdot} = -\nu^2 x + \sin \tau, \quad G(\tau) \equiv \nu^2 x - P < 0 \tag{1.3}$$

Here (1.1) are the equations of impact-free motions, (1.2) are the equations of the shock interactions, (1.3) are the equations of the possible state of the kinematic connection between mass m_1 and the barrier. By $y_-^{\cdot}$, $y_+^{\cdot}$ we denote the values of the dimensionless velocity $y^{\cdot}$ before and after, an instantaneous impact with a recovery factor $R \in [0, \; 1)$.

The system's phase space, formed by the coordinates $x, \; x^{\cdot}, \; \tau, \; y, \; y^{\cdot}$, is five-dimensional. Since in the equations of motion the variable τ occurs explicitly only as the argument of a 2π-periodic function, we identify the hyperplanes $\tau = \text{const}$ differing by $2\pi n$ (n is an integer). In the case of oscillations in a model with collisions the representative point $M \; (y, \; y^{\cdot}, \; x, \; x^{\cdot}, \; \tau)$, moving in the region $y > 0$ of the phase space, at some instant τ_0 hits onto the impact interaction halfsurface Π ($y = 0, \; y^{\cdot} < 0$) at a point $M_0 \; (0, \; y_0^{\cdot}, \; x_0, \; x_0^{\cdot}, \; \tau_0)$, where

$$y_0^{\cdot} = y_-^{\cdot} \; (\tau_0), \quad x_0 = x \, (\tau_0), \quad x_0^{\cdot} = x^{\cdot} \; (\tau_0) \tag{1.4}$$

Thereupon, it is instantaneously transferred in accordance with law (1.2) onto the halfsurface $y = 0, \; y^{\cdot} > 0$, whence it goes into the region $y > 0$ until at an instant τ_1

*) An analogous conclusion has been made on the basis of an experimental study of the dynamics of a vibrating hammer in [5].

it once again hits onto Π at point $M_1(0, y_1^{\cdot}, x_1, x_1^{\cdot}, \tau_1)$. The point mapping $M_1 = T(M_0)$, generated by such motions, of the halfsurface Π into itself is determined by an integration of differential equations (1.1) with initial conditions (1.4) and can be represented by the following system of transcendental relations:

$$\begin{aligned}
&\nu^2\varepsilon \sin(\sigma_0+\tau_0) - {}^1/_2 Pg(\sigma_0+\tau_0)^2 + r_1 + r_2\sigma_0 + r_3\cos\omega\sigma_0 + \\
&\quad r_4 \sin\omega\sigma_0 = 0 \\
&\mu y_1^{\cdot} - \nu^2\varepsilon\cos(\sigma_0+\tau_0) + Pg(\sigma_0+\tau_0) - r_2 + \omega(r_3\sin\omega\sigma_0 - \\
&\quad r_4\cos\omega\tau_0) = 0 \\
&(\mu-\nu^2)\varepsilon\sin(\sigma_0+\tau_0) + P[{}^1/_2 g(\sigma_0+\tau_0)^2 - \omega^{-2}] - \\
&\quad r_1 - r_2\sigma_0 + \mu(r_3\cos\omega\sigma_0 + r_4\sin\omega\sigma_0 + x_1) = 0 \\
&(\mu-\nu^2)\varepsilon\cos(\sigma_0+\tau_0) + Pg(\sigma_0+\tau_0) - r_2 - \\
&\quad \mu[\omega(r_3\sin\omega\sigma_0 - r_4\cos\omega\sigma_0) + x_1^{\cdot}] = 0
\end{aligned} \tag{1.5}$$

Here

$$\begin{aligned}
&g = \mu/(1+\mu), \quad \omega^2 = \nu^2/g, \quad \varepsilon = 1/(1-\omega^2) \\
&r_1 = g[x_0 - P(\mu^{-1}\omega^{-2} - {}^1/_2\tau_0^2) + \sin\tau_0] \\
&r_2 = g(x_0^{\cdot} - R\mu y_0^{\cdot} + P\tau_0 + \cos\tau_0) \\
&r_3 = g(P\mu^{-1}\omega^{-2} - \varepsilon\sin\tau_0 - x_0) \\
&r_4 = -g\omega^{-1}(x_0^{\cdot} + Ry_0^{\cdot} + \varepsilon\cos\tau_0)
\end{aligned}$$

$\sigma_0 = \tau_1 - \tau_0$ is the smallest simple positive root of the first equation in (1.5).

2. Derivation of the equations for the boundary of the region of sliding motions. When the representative point hits at instant τ_0 onto the sheet Π_s of sliding motions, situated on the halfsurface Π between the manifold $y^{\cdot}=0$ and a certain boundary Γ_s, its subsequent motion takes place by an infinite alternating sequence of impact — impact-free segments of the phase trajectory and ends at the exit point $M_s(0, 0, x_s, x_s^{\cdot}, \tau_s)$.

In the autonomous version of system (1.1) – (1.3), namely, $F = 0$ with $P_1 = P_2 = 0$ the question of an infinite-impact interaction of a two-mass system with a stop has been treated in [6] (for the case $y_0^{\cdot} = x_0^{\cdot}$) and in Example 3 in [1].

As the number i of collisions grows, the intervals $\sigma_i = \tau_{i+1} - \tau_i$ decrease, and the phase trajectory of the sliding regime approximates the trajectory, defined by relations (1.3), of the motion of a model with a superimposed kinematic constraint [2]. Here the necessary condition $y^{\cdot\cdot} < 0$ for the sliding regime differs all the less from condition, written in the same way, for the motion of a system in a kinematic constraint state (inequality (1.3)). Keeping the mentioned peculiarity in mind, a sliding regime can be idealized, at the expense of choosing its starting instant τ_0 with any degree of accuracy as the motion of a system with a superimposed kinematic connection between mass m_1 and a fixed barrier after an absolutely inelastic interaction [1, 2]. Such a method permits us to approximate M_s by a phase point corresponding to the termination of the model's motion in accordance with Eqs. (1.3), and, consequently, to determine the coordinates of the exit point from the conditions

$$G(\tau_s) = 0, \qquad G^{\cdot}(\tau_s) > 0 \tag{2.1}$$

Thus,

$$y_s = 0, \qquad y_s^{\cdot} = 0, \qquad x_s = P/\nu^2 \tag{2.2}$$

From (2.1), as a result of integrating differential equation (1.3) with initial conditions (1.4), we have the dependencies

$$\varphi_0 \equiv P - \nu^2 [(x_0 - \alpha \sin \tau_0) \cos \nu h_0 + \alpha \sin (\tau_0 + h_0)] - \nu (x_0^{\bullet} - \alpha \cos \tau_0) \sin \nu h_0 = 0 \quad (2.3)$$

$$x_s^{\bullet} = \nu (\alpha \sin \tau_0 - x_0) \sin \nu h_0 + (x_0^{\bullet} - \alpha \cos \tau_0) \cos \nu h_0 + \alpha \cos (\tau_0 + h_0) = 0, \ \alpha = (\nu^2 - 1)^{-1} \quad (2.4)$$

for finding the remaining coordinates $x_s^{\bullet}$, τ_s. Here $h_0 = \tau_s - \tau_0$ is the duration of the sliding state, defined as the smallest positive root of Eq. (2.3), satisfying the condition $x_s^{\bullet} > 0$. The transformation $M_0 \to M_s$ generated by relations (2.2) – (2.4) is denoted by S.

On the boundary manifold Γ_s, according to [1], the quantities σ_0 and h_0 are connected by the equality

$$\sigma_0 = \Theta h_0 \quad (2.5)$$

where the sliding state duration factor Θ is a known function of R. Equation (2.5) together with (2.3) and the first equation in (1.5) impose an additional constraint on the coordinates of point M_0 and, by the same token, permit us to determine in the first approximation the boundary Γ_s of the region of existence of the sliding regimes. Refined approximations to Γ_s can be obtained by examining the inverse point mappings $T^{-k} (\Gamma_s)$, as was done, for example, in [7]. Here only the number of the collision taken from the start of the sliding state, is shifted each time.

3. Equations of the fixed point. The sliding regime terminates at the exit point. The succeeding motion is effected along the phase trajectory, issuing from the exit point of impactless motions up to the instant τ' of next arrival of the trajectory on the halfsurface Π at the point M' $(0, y^{\bullet\prime}, x', x^{\bullet\prime}, \tau')$. Relations

$$\varphi_j = 0, \qquad j = 1, 2, 3, 4 \quad (3.1)$$

for computing the quantities $y^{\bullet\prime}$, x', $x^{\bullet\prime}$, $\sigma_s = \tau' - \tau_s$ coincide notationwise with the corresponding Eqs. (1.5) and are obtained from them by replacing the indices 0 and 1 by index s and a prime, respectively, in the phase coordinate notation. If $M' \in \Pi_s$ and $M' = M_0$, i.e.

$$y^{\bullet\prime} = y_0^{\bullet}, \ x' = x_0, \ x^{\bullet\prime} = x_0^{\bullet}, \ \tau' = \tau_0 \ (\mathrm{mod}\ 2\pi n) \quad (3.2)$$

then the motion of the model being examined is, on the whole, periodic and n-fold.

As is well known, the study of periodic processes reduces to the investigation of the fixed point of an appropriate point transformation. It is evident that a fixed point M_* of the approximating product ST of transformations corresponds to the motion being considered having a range of the sliding regime in the first approximation of the idealization adopted. By introducing the notation $y_*^{\bullet}$, x_*, $x_*^{\bullet}$, τ_*, for the coordinates of the fixed point M_* satisfying equalities (3.2), we obtain the following system of equations:

$$\nu^2 \varepsilon \sin \tau_* - {}^1/_2\, Pg (2\pi n + \tau_*)^2 + \alpha_1 + \alpha_2 \omega^{-1} \beta + \alpha_3 \cos \beta + \alpha_4 \sin \beta = 0 \quad (3.3)$$

$$\nu^2 \varepsilon \cos \tau_* - Pg (2\pi n + \tau_*) + \alpha_2 - \mu y_*^{\bullet} - \omega (\alpha_3 \sin \beta - \alpha_4 \cos \beta) = 0$$

$$(\mu - \nu^2)\,\varepsilon \sin \tau_* + P\,[{}^1/_2 g\,(2\pi n + \tau_*)^2 - \omega^{-2}] -$$
$$\alpha_1 - \alpha_2 \omega^{-1}\beta + \mu\,(x_* + \alpha_3 \cos\beta + \alpha_4 \sin\beta) = 0$$
$$\mu\,[x_*^{\cdot} - \omega\,(\alpha_3 \sin\beta - \alpha_4 \cos\beta)] + (\mu - \nu^2)\,\varepsilon \cos\tau_* +$$
$$Pg\,(2\pi n + \tau_*) - \alpha_2 = 0$$
$$\nu^2\,[(x_* - \alpha \sin\tau_*) \cos \nu h_* + \alpha \sin \tau_s] +$$
$$\nu\,(x_*^{\cdot} - \alpha \cos \tau_*) - P = 0$$

for determining them from relations (2.3), (2.4), (3.1). Here

$$\alpha_1 = g\,[P\,(\nu^{-2} - \mu^{-1}\omega^{-2} + {}^1/_2 \tau_s^2) + \sin\tau_s]$$
$$\alpha_2 = g\,(x_{s*}^{\cdot} + P\tau_s + \cos\tau_s), \qquad h_* = \tau_s - \tau_*$$
$$\alpha_3 = g\,[P\,(\mu^{-1}\omega^{-2} - \nu^{-2}) - \varepsilon \sin\tau_s]$$
$$\alpha_4 = -g\omega^{-1}(x_{s*}^{\cdot} + \varepsilon\cos\tau_s), \qquad \beta = \omega\,(2\pi n - h_*)$$
$$x_{s*}^{\cdot} = (x_*^{\cdot} - \alpha\cos\tau_s)\cos\nu h_* - \nu\,(x_* - \alpha\sin\tau_*)\sin \nu h_* + \alpha\cos\tau_s$$

4. Existence and stability of periodic motions with a sliding regime range. Under a continuous variation of parameters the fixed point M_*, and, consequently, the periodic motion with a sliding regime range, vanishes: (a) when the existence and stability conditions are violated, (b) because of the discontinuities in the point transformation ST, (c) as a consequence of the point M_* going onto the boundary of the region Π_s of sliding motions. We examine the cases listed in detail.

a) The limit value of the inequality

$$|z| < 1 \qquad (4.1)$$

where z is a root of the characteristic equation $X(z) = 0$, corresponds to a degeneration of the existence and stability conditions. Setting up the determinant

$$(j = 0, 1, 2, 3, 4)$$

$$X(z) = \begin{vmatrix} \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial\varphi_j}{\partial\tau'} z + \dfrac{\partial\varphi_j}{\partial\tau_0} & \dfrac{\partial\varphi_j}{\partial y'} z + \dfrac{\partial\varphi_j}{\partial y_0^{\cdot}} & \dfrac{\partial\varphi_j}{\partial x'} z + \dfrac{\partial\varphi_j}{\partial x_0} & \dfrac{\partial\varphi_j}{\partial x^{\cdot\prime}} z + \dfrac{\partial\varphi_j}{\partial x_0^{\cdot}} & \dfrac{\partial\varphi_j}{\partial\tau_s} \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

and expanding it at the point M_*, after simplifying manipulations we obtain the second-order characteristic equation in the following parametric form:

$$a_0 z^2 + a_1 z + a_2 = 0$$

Here

$$a_0 = -\mu y_*^{\cdot}, \quad a_1 = (\mu y_*^{\cdot} + x_*^{\cdot})\,[\delta\,\nu \sin \nu h_* -$$
$$g\,(1 - \cos\beta)\cos\nu h_*] - [\delta \cos\nu h_* + g\,(1 - \cos\beta)\,\nu^{-1}\sin\nu h_*] -$$
$$\mu y_*^{\cdot}\,\{\nu\omega^{-1}(\beta + \sin\beta)\sin\nu h_* - [g\,(\mu^{-1} + \cos\beta) + 1]\cos\nu h_*\}$$
$$a_2 = g\,[\omega^{-1}(\gamma + \nu^2 x_* - \sin\tau_*)\,(\beta\cos\beta - \sin\beta)] -$$
$$g y_*^{\cdot}\,(\mu + \mu^{-1} + 2\cos\beta + \beta\sin\beta), \quad \delta = g\omega^{-1}(\beta - \sin\beta)$$
$$\gamma = (\mu - \nu^2)\,\mu^{-1}\varepsilon\sin\tau_* -$$
$$g\,[P\mu^{-1} + \omega\,(x_{s*}^{\cdot} + \varepsilon\cos\tau_s)\sin\beta + (P + \omega^2\varepsilon\sin\tau_s)\cos\beta]$$

As is well known, the stability region is delineated by the following conditions, equivalent to inequality (4.1):

$$|a_2| < |a_0|, \qquad |a_1| < |a_0 + a_2| \qquad (4.2)$$

In the limit, relations (4.2) together with (3.3) determine the bifurcation N-surfaces of existence and stability.

b) A violation of the continuity of the point transformation ST is connected with the appearance at a certain instant $\tau^* \in (\tau_s, \tau_*)$ of an additional interaction of mass m_1 with the fixed barrier. The boundary surface C associated with such a degeneracy is determined by the analytic conditions of tangency of the phase trajectory with the halfsurface Π

$$y(\tau^*) = 0, \qquad y^{\cdot}(\tau^*) = 0 \qquad (4.3)$$

Thus, the system of equations of manifold C consists of (3.3), (4.3). In developed form the right-hand sides of equalities (4.3) coincide, to within the notation for the instant of tangency, with the expressions for φ_1 and φ_2.

c) Finally, a degeneracy of infinite-impact periodic motions in the oscillations of a system with a finite number of impacts over a period [1, 7] takes place on the last part of the boundary surfaces, e.g., the manifolds C_s. The equations of connection between the parameters, which together with (3.3) define the boundary C_s, are obtained from relations (1.5), (2.3), (2.5), written in the coordinates of the fixed point, as the limit condition $M_* \in \Gamma_s$ of the constraint $M_* \in \Pi_s$.

The construction of the boundaries of the region of existence of stable periodic motions with a sliding regime range in the space of parameters ν, P, μ, R was carried out by means of a numerical investigation of the corresponding equations. The required values of coefficient Θ were determined both from the approximate relations in [1] as well as from the exact formula for the linear law of variation of $y^{\cdot\cdot}(\tau)$ at the terminal stage of the sliding state

$$\Theta^3 - 5\Theta^2 + 5\Theta + (1 - R)(2\Theta - 3) = 0, \quad \Theta \in (0,1] \qquad (4.4)$$

This is a limit dependency for the asymptotic representation of Θ [1] and follows directly from the conditions of existence of the invariant curve $y^{\cdot} = \gamma_s\tau^2 (\gamma_s = \Theta(\Theta - 3) / 6R)$ for the corresponding point mapping (*).

Figures 1, 2 show individual sections D_s of the region of existence and stability of single ($n = 1$) periodic motions on surfaces ν = const, μ = const. Boundaries *1*—*4* (Fig. 1) were constructed for $\nu = 0.9$ and for the following values of μ: *1* for $\mu = 5$, *2* for $\mu = 2$, *3* for $\mu = 1$, *4* for $\mu = 0.4$. The boundary manifolds *5*—*8* (Fig. 2) correspond to the parameter value sets: *5* for $\nu = 0.8$, $\mu = 1$; *6* for $\nu = 0.8$, $\mu = 0.4$; *7* for $\nu = 0.7$, $\mu = 0.4$; *8* for $\nu = 0.7$, $\mu = 0.2$. The existence and stability boundaries are delineated by the shading; here the segment shaded twice is a part of the N-boundary not depending on μ. For the actual values of ν and μ the section D_s is the region contained between the corresponding N-boundaries and the manifold C_s (the thin lines). In the cases being considered the curves C do not occur in the composition of the boundaries of the existence and stability regions.

In Fig. 1 the dashed lines show the boundaries of the existence region for $\mu = 5$:

*) Relation (4.4) can be obtained also from the equations of motion for a boundary process in the generating approximation [8].

N_0, $N_{2\pi}$, C_s. As P varies continuously from N_0 to $N_{2\pi}$ the sliding state time h_* grows from zero to 2π. However, as we see from the graph, a significant part of the existence region is eliminated because of loss of stability. Thus, it follows from the results obtained that stable periodic motions with a sliding state part can be realized in sufficiently large regions of the parameter space, expanding as μ decreases and as $\nu \rightarrow 1$; in the latter case the sections D_s are displaced upwards along the P-axis. In contrast to models with one degree of freedom a part of the existence regions in two-degrees systems can be cut off by the stability boundaries for oscillations of the type being examined.

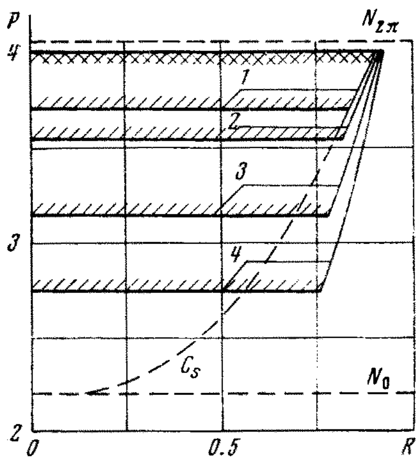

Fig. 1

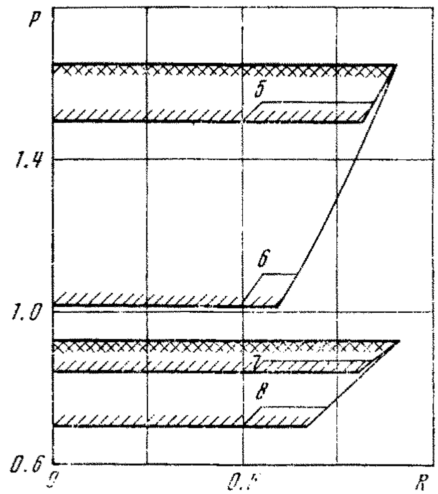

Fig. 2

By virtue of the approximations, justified in [1], for an infinite-shock converging process we see that in the first approximation the dynamic characteristics of the periodic motions with a sliding regime range do not depend on R. Thus, when calculating and designing two-mass shock-oscillatory systems under conditions of changing values of the velocity recovery factor, an oscillatory mode of the type studied can be assigned as the law of motion in order to ensure the stability of the operation.

The basic results obtained in this paper were verified by simulating the equations of motion of system (1.1) – (1.3) on an electronic computer, which permitted us to restrict ourselves to solving the problem in the first approximation.

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ON THE CORRECTNESS OF THE APPROXIMATE INVESTIGATION OF SYNCHRONOUS MACHINE ROTOR SWINGING

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We consider the complete system of equations for the dynamics of a synchronous machine with two windings on the rotor. We indicate the conditions under which the original system of equations can be reduced to the equation of motion of the rotor. The conditions for rotor selfoscillations to arise are determined as a result of investigating this equation. The complete system of equations for the dynamics of a synchronous machine containing equations describing the electrical responses and equations for the rotor's mechanical motion are obtained in [1]. Transient responses in electric circuits were investigated next, as was the expression for the electromechanical moment under a constant rotation velocity of a rotor with one circuit, e. g., field winding. However, in many of the later works the electrical equations were used only for finding the electromechanical moment under a constant spin rate of the rotor, and the problem was then reduced to the study of the equation for the rotor's mechanical motion [2, 3]. Here the conditions for which such an analysis is admissible were not mentioned. It was established that the swinging of a synchronous machine's rotor can be revealed in the form of selfoscillations. Vlasov [4] has investigated the equation of motion of a rotor and, under the assumption of a small parameter in the first derivative term, has found the conditions for the excitation of selfoscillations. Investigation in this same direction was carried out in [5]. However, in the investigation of the selfoscillations Vlasov did not examine the responses in the electrical circuits, while the expression for the electromechanical moment was obtained from power considerations. Other works have used particular expressions for the electromechanical moment, which can not explain the selfoscillation phenomenon.

1. Equations for synchronous machine dynamics. The equations for the dynamics of a salient-pole synchronous machine with two rotor windings — the